Adaptive Control

Chapter 11: Direct Adaptive Control

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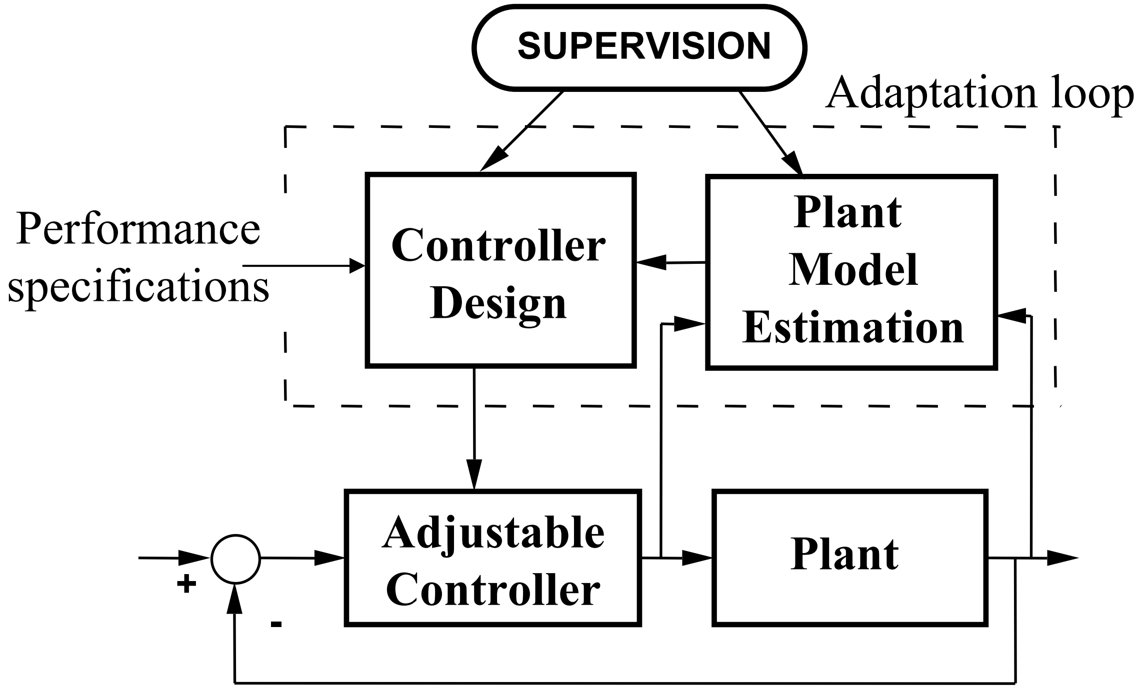
Abstract Direct adaptive control covers the schemes where the parameters of the controller are directly updated from a signal error (adaption error) reflecting the performance error. The chapter presents strategies for direct adaptive control in a deterministic and in a stochastic environment and the corresponding analysis. This includes adaptive tracking and regulation with independent objectives, adaptive minimum variance control and their extensions. Robustification of direct adaptive control schemes is also discussed.

Adaptive Control – A Basic Scheme



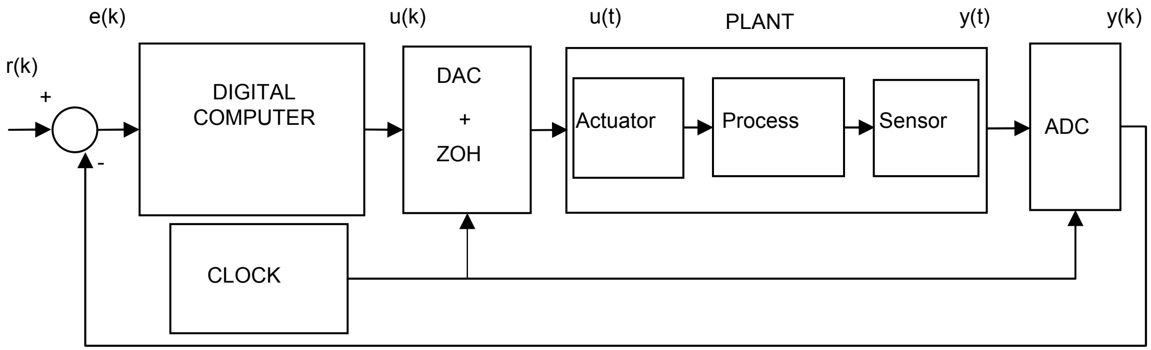
- Indirect adaptive control
- Direct adaptive control *(the controller is directly estimated)*

Outline

- Digital control systems
- Tracking and regulation with independent objectives (known parameters)
- Adaptive tracking and regulation with independent objectives (direct adaptive control)
- Pole placement (known parameters)
- Adaptive pole placement (indirect adaptive control)

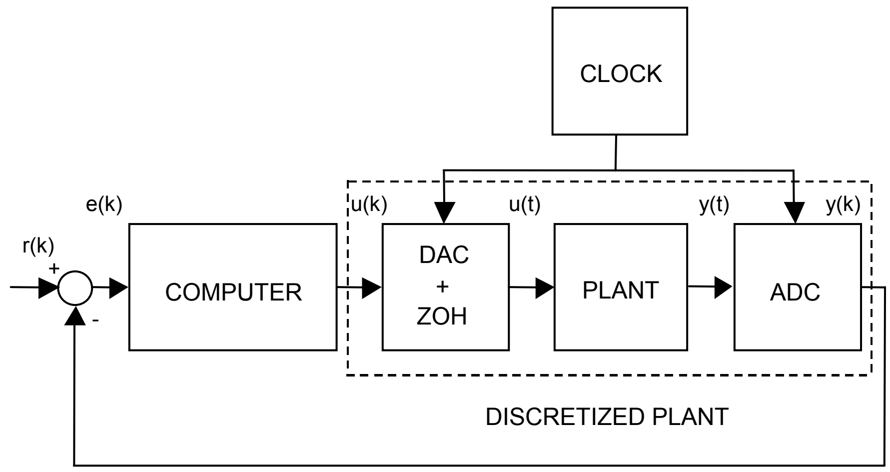
Digital Control System

The *control law* is implemented on a digital computer



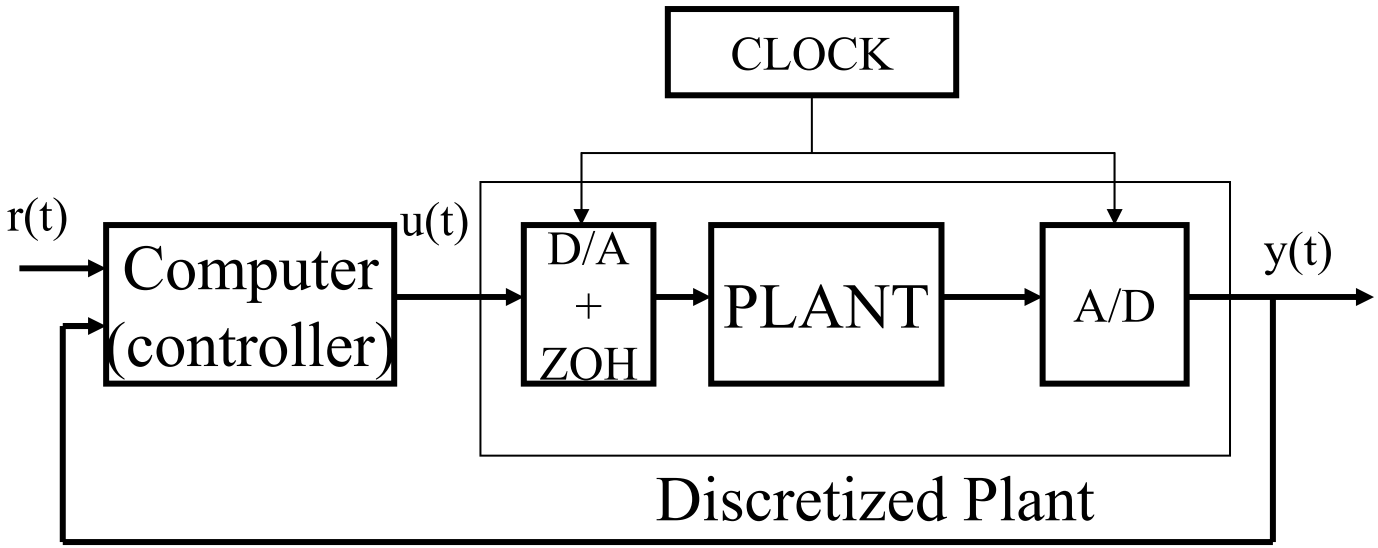
ADC: analog to digital converter
DAC: digital to analog converter
ZOH: zero order hold

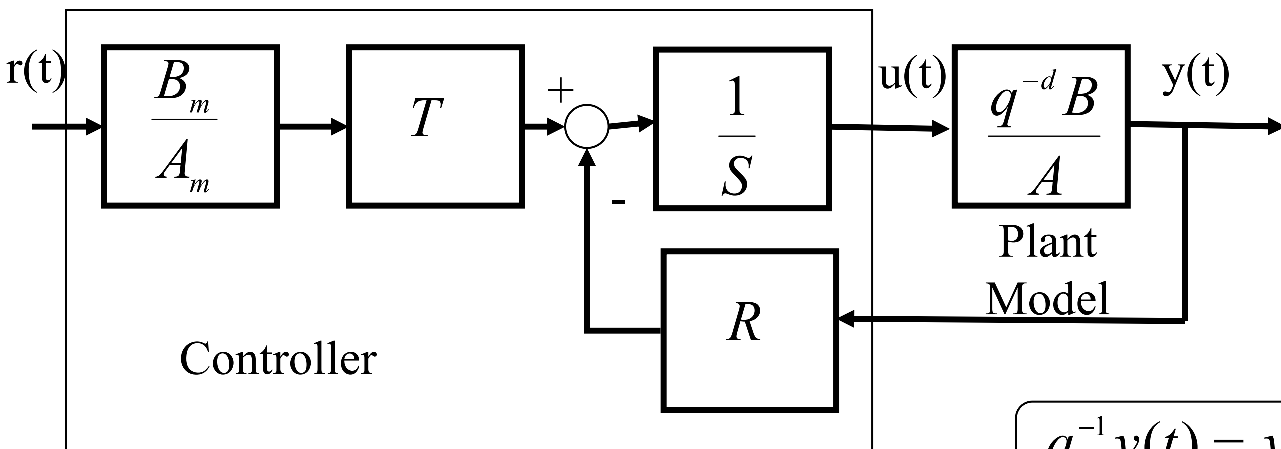
Digital Control System



- Sampling time depends on the system bandwidth
- Efficient use of computer resources

The R-S-T Digital Controller





$$q^{-1}y(t) = y(t-1)$$

Discrete time model – *General form*

$$(*)\quad y(t) = -\sum_{i=1}^{n_A} a_i y(t-i) + \sum_{i=1}^{n_B} b_i u(t-d-i)$$

d –delay (integer multiple of the sampling period)

$$1+\sum_{i=1}^{n_A} a_i q^{-i} = A(q^{-1}) = 1 + q^{-1}A^*(q^{-1}) \;;\quad A^*(q^{-1}) = a_1 + a_2 q^{-1} + \ldots + a_{n_A} q^{-n_A+1}$$

$$\sum_{i=1}^{n_B} b_i q^{-i} = B(q^{-1}) = q^{-1}B^*(q^{-1}) \;;\quad B^*(q^{-1}) = b_1 + b_2 q^{-1} + \ldots + b_{n_B} q^{-n_B+1}$$

$$(*)\; A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t)$$

$$(*)\; A(q^{-1})y(t+d) = B(q^{-1})u(t)$$ (Predictive form)

$$(*)\; y(t) = H(q^{-1})u(t) \;;\quad H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$ - pulse transfer operator

$$q^{-1} \to z^{-1} \qquad H(z^{-1}) = \frac{q^{-z}B(z^{-1})}{A(z^{-1})}$$ - transfer function

First order systems with delay

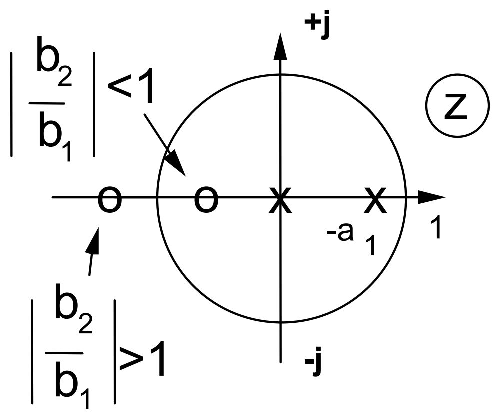
Fractional delay

Continuous time model $H(s) = \dfrac{G e^{-s\tau}}{1+T_s}$ $\qquad \tau = d.T_s + L\,;\ 0 < L < T_s$

Discrete time model $$H(z^{-1}) = \frac{z^{-d}(b_1 z^{-1} + b_2 z^{-2})}{1 + a_1 z^{-1}} = \frac{z^{-d-1}(b_1 + b_2 z^{-1})}{1 + a_1 z^{-1}}$$

$$a_1 = -e^{-\frac{T_s}{T}} \qquad b_1 = G(1 - e^{\frac{L-T_s}{T}}) \qquad b_2 = G e^{-\frac{T_s}{T}}(e^{\frac{L}{T}} - 1)$$

Remark: For $L > 0.5T_s \Rightarrow b_2 > b_1 \Rightarrow unstable \quad zero \; (\left|-\dfrac{b_2}{b_1}\right| > 1)$



o - zero
x - pole

Tracking and regulation with independent objectives

It is a particular case of pole placement
(the closed loop poles contain the plant zeros))

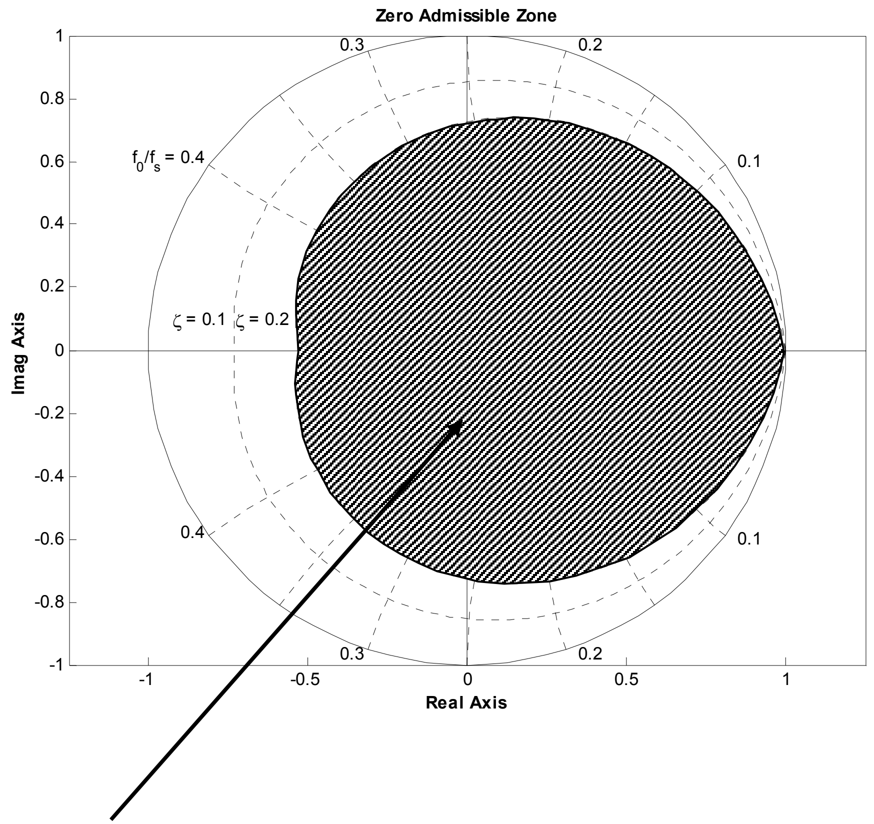
It is a method which simplifies the plant zeros
Allows exact achievement of imposed performances

Allows to design a RST controller for:
- stable or unstable systems
- without restrictions upon the degrees of the polynomials A et B
- without restriction upon the integer delay d of the plant model
- discrete-time plant models with *stable zeros!!!*

Remarks:
- *Does not tolerate fractional delay > 0.5* T_S *(unstable zero)*
- High sampling frequency generates unstable discrete time zeros !

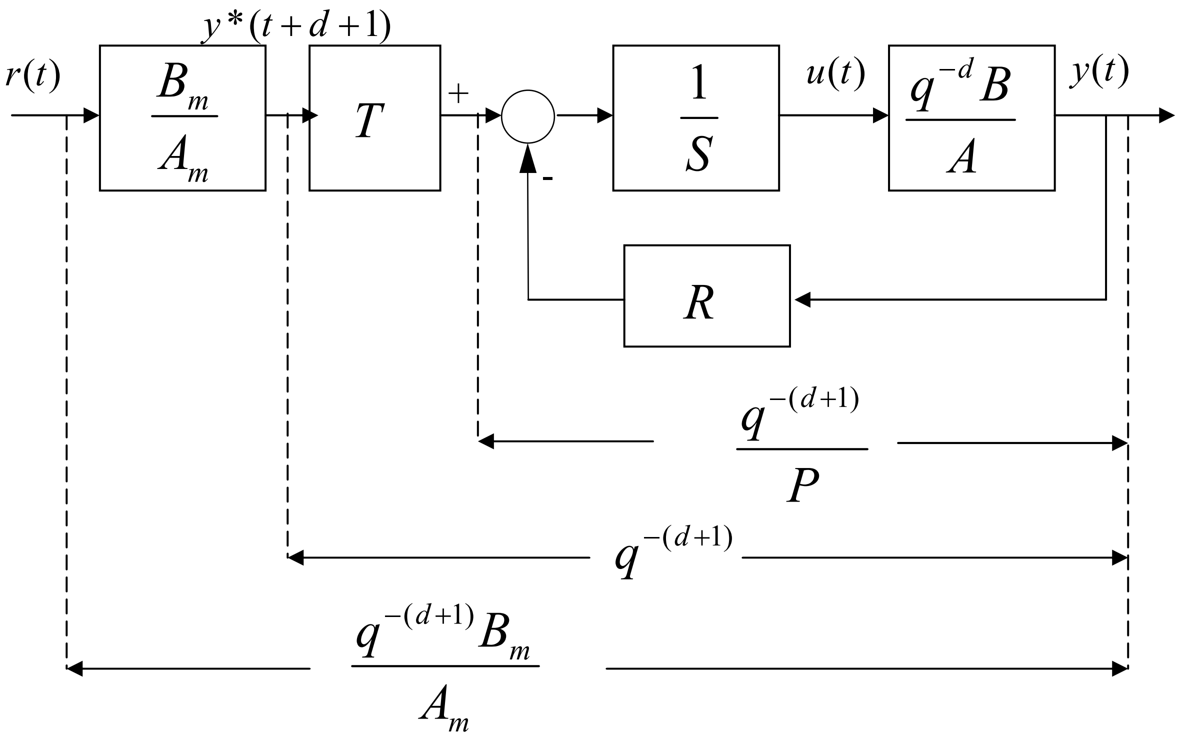
Tracking and regulation with independent objectives

The model zeros should be stable and enough damped



Admissibility domain for the zeros of the discrete time model

Tracking and regulation with independent objectives



$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Reference signal (tracking): $y^*(t+d+1) = \dfrac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$

Controller: $S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)$

Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

T.F. of the closed loop without *T:*

$$H_{CL}(q^{-1}) = \frac{q^{-d+1}B^*(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1})} = \frac{q^{-d+1}}{P(q^{-1})} = \frac{q^{-d+1}B^*(q^{-1})}{B^*(q^{-1})P(q^{-1})}$$

The following equation has to be solved :

$$A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1}) = B^*(q^{-1})P(q^{-1}) \quad (*)$$

S should be in the form: $S(q^{-1}) = s_0 + s_1 q^{-1} + \ldots + s_{n_S} q^{-n_S} = B^*(q^{-1})S'(q^{-1})$

After simplification by B^*,(*) becomes:

$$\boxed{A(q^{-1})S'(q^{-1}) + q^{-d+1}R(q^{-1}) = P(q^{-1})} \qquad (**)$$

Unique solution if: $n_P = deg\, P(q^{-1}) = n_A + d$; $\boxed{deg\, S'(q^{-1}) = d}$; $deg\, R(q^{-1}) = n_A - 1$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots r_{n_A-1} q^{-n_A-1} \qquad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_d q^{-d}$$

Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

(**) is written as: $Mx = p \longrightarrow x = M^{-1}p$

$$
M = \underbrace{\left[\overbrace{\begin{array}{cccccccc}
1 & 0 & & & & & & 0 \\
a_1 & 1 & & & & & & . \\
a_2 & a_1 & & 0 & & & & . \\
: & : & & 1 & & & & . \\
a_d & a_{d-1} & \dots & a_1 & 1 & & & . \\
a_{d+1} & a_d & & & a_1 & 1 & & . \\
a_{d+2} & a_{d+1} & & & a_2 & 0 & & . \\
 & & & & . & & & . \\
 & & & & . & & & 0 \\
0 & 0 & \dots & 0 & a_{n_A} & 0 & 0 & 1
\end{array}}^{n_A + d + 1}\right]}_{d+1 \qquad\qquad n_A} \quad \Big\} \; n_A + d + 1
$$

$x^T = [1, s'_1, \dots, s'_d, r_0, r_1, \dots, r_{n-1}]$ $\qquad$ $p^T = [1, p_1, p_2, \dots, p_{n_A}, p_{n_A+1}, \dots, p_{n_A+d}]$

Use of WinReg or *predisol.sci(.m)* for solving (**)

Insertion of pre specified parts in R and S is possible

Tracking (computation of $T(q^{-1})$)

Closed loop T.F.: $r \longrightarrow y$

$$H_{BF}(q^{-1}) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})} = \frac{B_m(q^{-1})T(q^{-1})q^{-(d+1)}}{A_m(q^{-1})P(q^{-1})}$$

Desired T.F.

It results : $T(q^{-1}) = P(q^{-1})$

Controller equation:

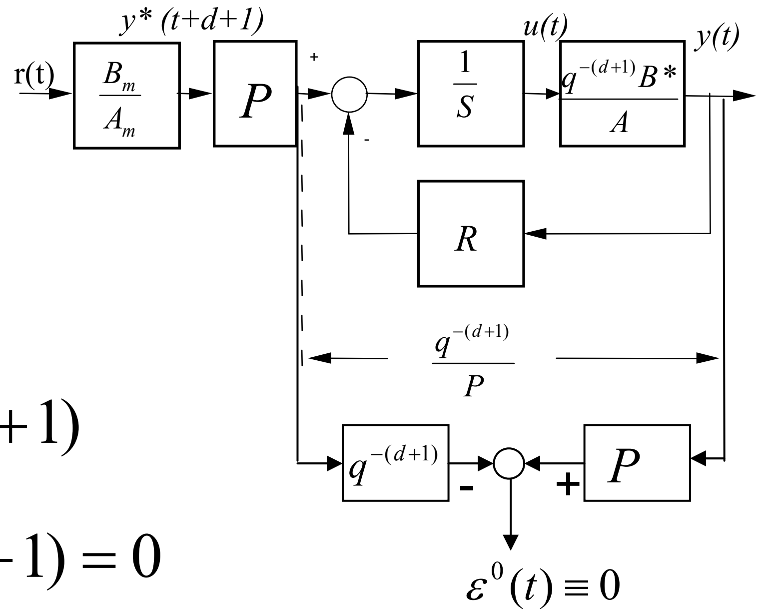
$$S(q^{-1})u(t) = P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)$$

$$u(t) = \frac{P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$u(t) = \frac{1}{b_1}\left[P(q^{-1})y^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)\right] \quad (s_0 = b_1)$$

Reference signal (tracking) : $y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(t)$

Tracking and regulation with independent objectives
A time domain interpretation



(in case of correct tuning)

$$y(t) = \frac{q^{-(d+1)}}{P(q^{-1})} y^*(t+d+1)$$

$$P(q^{-1}) y(t) = q^{-(d+1)} y^*(t+d+1)$$

$$P(q^{-1}) y(t) - q^{-(d+1)} y^*(t+d+1) = 0$$

Reformulation of the "design problem":

Find a controller which generate *u(t)* such that:

$$\varepsilon^0(t+d+1) = P\left[y(t+d+1) - y^*(t+d+1)\right] = 0$$

Tracking and regulation with independent objectives
Synthesis in the time domain – an example

For *d=0 (S'=1)*

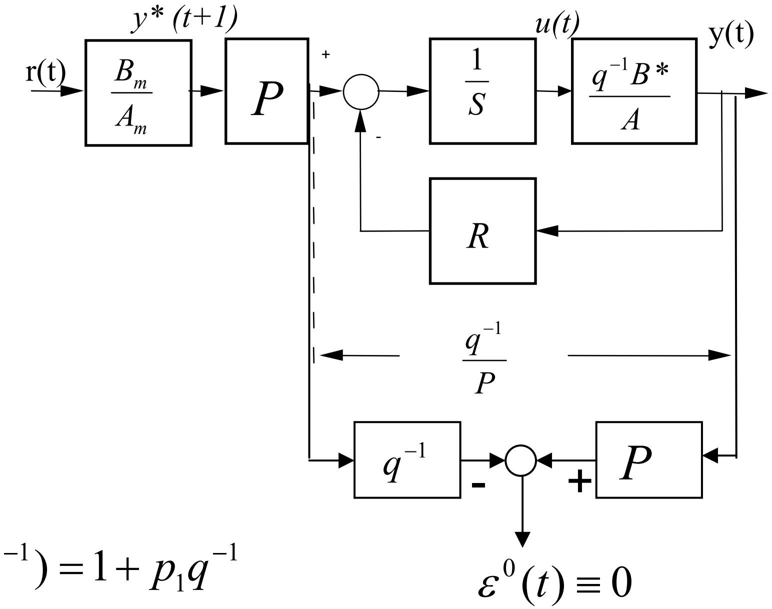
$$S(q^{-1}) = B^*(q^{-1})$$

$$A(q^{-1}) + q^{-1}R(q^{-1}) = P(q^{-1})$$

$$R(q^{-1}) = P^*(q^{-1}) - A^*(q^{-1})$$

$$P(q^{-1}) = 1 + q^{-1}P^*(q^{-1})$$

$$A(q^{-1}) = 1 + q^{-1}A^*(q^{-1})$$



Example:

$$y(t+1) = -a_1 y(t) + b_1 u(t) + b_2 u(t-1); \quad P(q^{-1}) = 1 + p_1 q^{-1}$$

$$\varepsilon^\circ(t+1) = P\left[y(t+1) - y^*(t+1)\right] = y(t+1) + p_1 y(t) - Py^*(t+1) =$$

$$= \left[-a_1 y(t) + b_1 u(t) + b_2 u(t-1) + p_1 y(t) - Py^*(t+1)\right] = 0$$ ← Solve for u(t)

$$u(t) = \frac{Py^*(t+1) - b_2 u(t-1) - r_0 y(t)}{b_1}; \qquad r_0 = p_1 - a_1$$

Controller satisfies:

$$P(q^{-1})y^*(t+1) = b_1 u(t) + b_2 u(t-1) + r_0 y(t) = \theta^T \phi(t)$$

$$\theta^T = [b_1, b_2, r_0] \ \ \phi^T(t) = [u(t), u(t-1), y(t)]$$

Adaptive tracking and regulation with independent objectives

Three techniques:

- Model reference adaptive control (direct)
- Plant model estimation + computation of the controller (indirect)
- Re-parametrized plant model estimation (direct)

Model Reference Adaptive Control

Objective: $$\lim_{t\to\infty}\varepsilon^0(t+1)=\lim_{t\to\infty}P(q^{-1})\left[y(t+1)-y^*(t+1)\right]=0$$

Adjustable controller: $$u(t)=\frac{Py^*(t+1)-\hat{b}_2(t)u(t-1)-\hat{r}_0(t)y(t)}{\hat{b}_1(t)}$$

$$P(q^{-1})y^*(t+1)=\hat{\theta}^T(t)\phi(t)$$

$$\hat{\theta}^T(t)=\left[\hat{b}_1(t),\hat{b}_2(t),\hat{r}_0(t)\right];\ \phi^T(t)=\left[u(t),u(t-1),y(t)\right]$$

But for the correct values of controller parameters one has:

$$P(q^{-1})y(t+1)=P(q^{-1})y^*(t+1)=\theta^T\phi(t)$$

And therefore one has: $$\varepsilon^0(t+1)=\left[\theta-\hat{\theta}(t)\right]^T\phi(t)$$

Define the a posteriori adaptation error: $$\varepsilon(t+1)=\left[\theta-\hat{\theta}(t+1)\right]^T\phi(t)$$

Use P.A.A.

However one should show in addition that $\|\phi(t)\|$ is bounded (i.e. plant input and output are bounded)

Plant model estimation + computation of the controller (indirect)

Step 1 : Plant model estimation

Plant model (unknown): $y(t+1) = -a_1 y(t) + b_1 u(t) + b_2 u(t-1) = \theta_P^T \phi(t)$

Adjustable predictor:

$$\hat{y}^0(t+1) = -\hat{a}_1(t) y(t) + \hat{b}_1(t) u(t) + \hat{b}_2(t) u(t-1) = \hat{\theta}_P^T(t)\phi(t)$$

$$\hat{\theta}_P^T(t) = \left[\hat{b}_1(t), \hat{b}_2(t), -\hat{a}_1(t)\right]; \phi^T(t) = \left[u(t), u(t-1), y(t)\right]$$

a priori prediction error: $\varepsilon^0(t+1) = y(t+1) - \hat{y}^0(t+1) = \left[\theta_P - \hat{\theta}_P(t)\right]^T \phi(t)$

a posteriori prediction error: $\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = \left[\theta_P - \hat{\theta}_P(t+1)\right]^T \phi(t)$

Use PAA

Step 2 : Computation of the controller

Compute at each instant t : $\hat{r}_0(t) = p_1 - \hat{a}_1(t)$

Adjustable controller: $P(q^{-1}) y^*(t+1) = \hat{\theta}^T(t)\phi(t)$

$$\phi^T(t) = \left[u(t), u(t-1), y(t)\right]; \hat{\theta}^T(t) = \left[\hat{b}_1(t), \hat{b}_2(t), \hat{r}_0(t)\right]$$

From $\hat{\theta}_P(t)$

In the general case d > 0 one will have to solve equation (**)

Re-parametrized plant model estimation (direct)

Plant model (unknown):

$$y(t+1) = -a_1 y(t) + b_1 u(t) + b_2 u(t-1) \pm p_1 y(t)$$

$$= -p_1 y(t) + \underbrace{(p_1 - a_1)}_{r_0} y(t) + b_1 u(t) + b_2 u(t-1) = -p_1 y(t) + \theta^T \phi(t)$$

Re-parametrized adjustable predictor:

$$\hat{y}^0(t+1) = -p_1 y(t) + \hat{r}_0(t) y(t) + \hat{b}_1(t) u(t) + \hat{b}_2(t) u(t-1)$$

$$= -p_1 y(t) + \hat{\theta}^T(t) \phi(t)$$

$$\hat{\theta}^T(t) = \left[\hat{b}_1(t), \hat{b}_2(t), r_0(t)\right]; \ \phi^T(t) = \left[u(t), u(t-1), y(t)\right]$$

a priori prediction error: $\varepsilon^0(t+1) = y(t+1) - \hat{y}^0(t+1) = \left[\theta - \hat{\theta}(t)\right]^T \phi(t)$

a posteriori prediction error: $\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = \left[\theta - \hat{\theta}(t+1)\right]^T \phi(t)$

Use PAA

One estimates directly the parameters of the controller

One has:

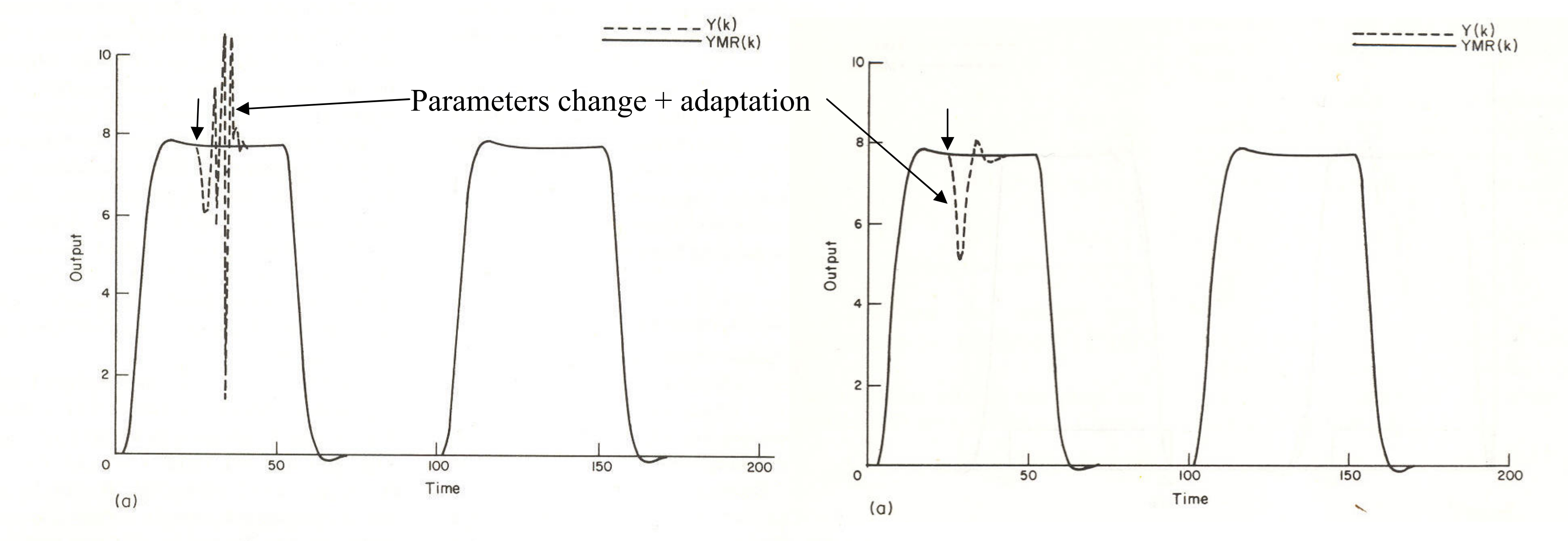
$$\lim_{t\to\infty} \varepsilon^0(t+1) = \lim_{t\to\infty} P(q^{-1})\left[y(t+1) - y^*(t+1)\right] = 0$$

Adaptive tracking and regulation with independent objectives

- Easy generalization for the case $d > 0$
- Elegant and simple solution for adaptation (direct)
- **Unfortunately restricted use in practice because it requires that the plant zeros remains always stable and well damped**

Direct Adaptive Control – Simulations results

Tracking

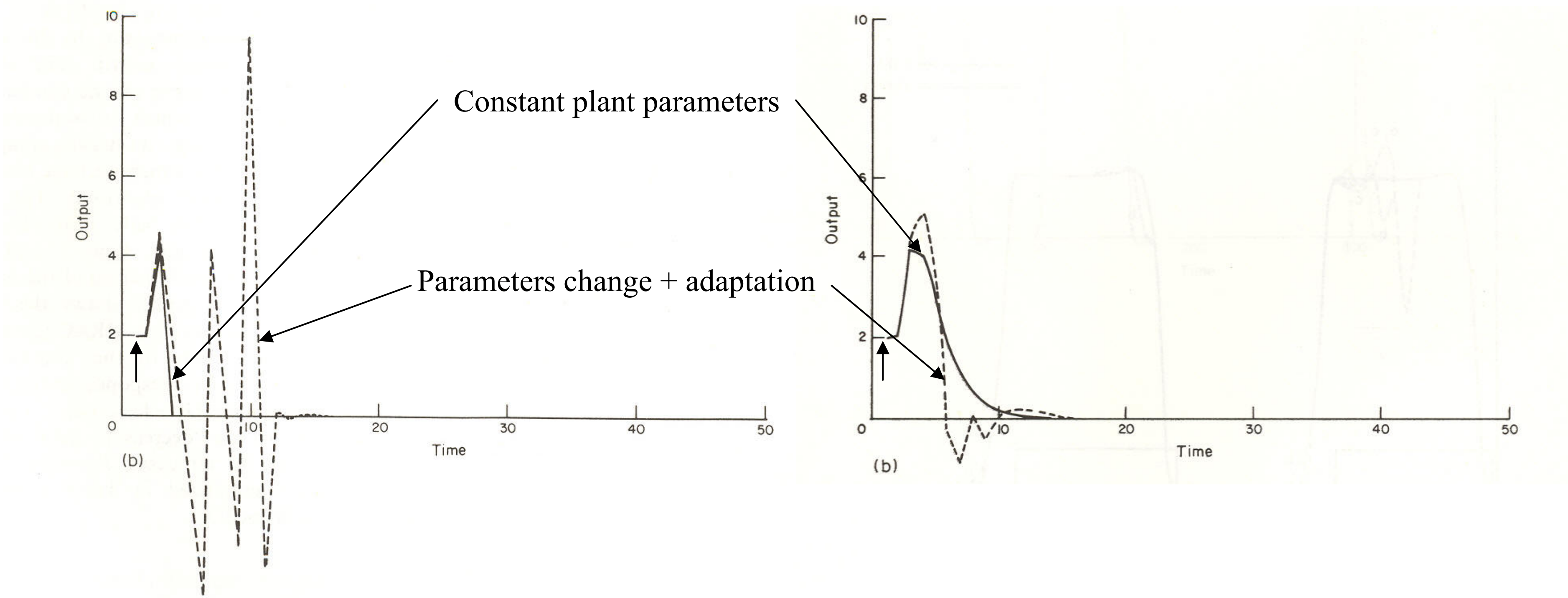


$$P(q^{-1}) = 1 \qquad\qquad P(q^{-1}) = (1 - 0.4q^{-1})$$

The choice of the poles for the closed loop (regulation) has a great influence upon adaptation transient behavior!

Direct Adaptive Control – Simulations results

Regulation



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